成绩

中国农业大学

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论文题目：<u>**Research Proposal: Solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential**</u>

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Abstract

The single Schrödinger model in soliton theory is studied widely by researchers. However, the physical phenomena that a single model can describe are limited. Now, due to the coupled Schrödinger models having important and wide application value in multicomponent Bose-Einstein condensate, solitons in the two-coupled Schrödinger model with parity-time-symmetric potential are investigated. In this work, modified squared-operator and fourth-order Runge-Kutta methods will be carried out to investigate the dynamic behavior and stability of solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential. It is expected to give the system parameters interval range for the presence of stable solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential. This work provides a new idea for the rational use of external potential control to obtain stable nonlinear modes, which can help researchers predict new physical phenomena and further guide practice.

Keywords: Solitons, Bose-Einstein condensates, parity-time-symmetric Scarf-II potential

Table Contents

Abstract 1
1. Introduction 3
2. Review of Relevant Literature 3
    2.1. Solitons 3
    2.2. Two-component Bose-Einstein Condensates 4
        2.2.1. The Definition of Bose-Einstein Condensates 4
        2.2.2. Multi-component Bose-Einstein Condensates 4
    2.3. Parity-time-symmetry 5
        2.3.1. Parity-time-symmetric Potential 6
        2.3.2. Scarf-II Potential 6
    2.4. Solitons in Bose-Einstein Condensates 6
    2.5. Solitons in Parity-time-symmetric System 7
3. Research Questions 7
4. Research Methods 8
5. Research Procedures 8
6. Conclusion 9
References (SpringerMathPhysNumber) 10
Appendix: Grading Sheet of Research Proposal 12

Research Proposal: Solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential

1. Introduction

This proposal outlines plans for research that I am considering undertaking for my Bachelor of Science in semester 2, 2023-2024. My proposed research is the dynamic behavior and stability of solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential in physical and mathematical areas by soliton theory and numerical methods [1-3].

The single Schrödinger model in soliton theory is studied widely by researchers. However, the physical phenomena that a single model can describe are limited. Due to the coupled Schrödinger models having important and wide application value in multicomponent Bose-Einstein condensate, solitons in the two-coupled Schrödinger model with parity-time-symmetric potential are investigated. The purpose of this study is to investigate the conditions for the existence of stable solitons in two-component Bose-Einstein condensates. First, the literature surveys are carried through the Web of Science database with a few keywords, such as “multicomponent Bose-Einstein condensate”, “coupled Schrödinger model”, “PT-symmetric potential”, and “soliton”. Then, the research questions, methods, and procedures will be introduced. Finally, the conclusion will be concluded.

2. Review of Relevant Literature

2.1. Solitons

Soliton is a significant phenomenon in nature. In mathematics and physics, a soliton is a nonlinear, self-reinforcing, strongly stable, localized wave packet [4]. Soliton can preserve its shape while propagating freely, at a constant velocity, and recover its shape even after collisions with other such localized wave packets [5]. The formation of the stable soliton is due to the balance of nonlinear and dispersive effects in the mediums [6]. Moreover, solitons are solutions of a wide class of weakly nonlinear dispersive partial differential equations describing physical systems, which provided us with the possibility to study solitons through mathematical methods [6]. Solitons are shown to exist in many nonlinear systems, for instance, nonlinear Schrödinger equations, Gross-Pitaevskii equations, and Ginzburg-Landau equations [7-9]. Research on solitons helps us better understand nature.

2.2. Two-component Bose-Einstein Condensates

2.2.1. The Definition of Bose-Einstein Condensates

Bose-Einstein condensate (BEC), as one of the important physical phenomena, attracts the attention of researchers. At extremely low temperatures, ultracold atoms are anticipated to condense into a collective state as their wave functions overlap, giving rise to the formation of a Bose-Einstein condensate, wherein atoms condense into the ground state of the wave function [10]. In condensed matter physics, BEC is a state of matter that is typically formed when a gas of bosons at very low densities is cooled to temperatures very close to absolute zero [11]. The successful observation of BEC becomes one of the research focuses in the fields of condensed matter physics and atom optics [12]. Generally, BECs can be realized by using atoms, molecules, quasi-particles, and photons [13]. Thus, research on BEC plays an important role in the development of theoretical physics.

2.2.2. Multi-component Bose-Einstein Condensates

Two-component or even multi-component BECs have significance in theoretical physics. Compared with the single-component BECs, the multi-component BECs possess inter-component interactions and have complex quantum phases and properties [14]. As one kind of multi-component BECs, the two-component BECs trapped in a quasi-one-dimensional harmonic potential at zero temperature can be described by the following coupled Gross-Pitaevskii equations [15]:

$$\mathrm{i}\hbar\frac{\partial\Psi_1}{\partial t}=\left(-\frac{\hbar^2}{2M}\nabla^2+V_1(\boldsymbol{r})+g_{11}|\Psi_1|^2+g_{12}|\Psi_2|^2\right)\Psi_1,$$

$$i\hbar\frac{\partial\Psi_2}{\partial t}=\left(-\frac{\hbar^2}{2M}\nabla^2+V_2(\boldsymbol{r})+g_{21}|\Psi_1|^2+g_{22}|\Psi_2|^2\right)\Psi_2,$$

where Ψ_j are the two-component parameters, $\boldsymbol{r}=(x,y,z)$, $\hbar$ is the Planck constant, M is the atomic mass, ∇^2 is the Laplacian, $V_j(\boldsymbol{r}) = [\omega_{jx}^2 x^2/2 + \omega_{j\perp}^2 (y^2 + z^2)/2]M/2$ stands for the harmonic potentials, g_{jj} are the interactions between atoms, and $g_{j,3-j}$ describe the inter-component interactions $(j = 1,2)$ [16]. However, the coupled Gross-Pitaevskii equations are not applicable, for instance, for the condensates of excitons, magnons, and photons, where the critical temperature is comparable to room temperature [17]. It is worth noting that the validity of this approach actually fits well for most alkali atoms experiments [14]. It is meaningful to investigate the two-component BECs in experiments.

Consider a situation in which the Gross-Pitaevskii equations can be simplified. If the trap frequencies in the radial directions $\omega_\perp$ are larger than the axial directions ω_{jx}, the coupled Gross-Pitaevskii equations become quasi-one-dimensional systems along the x direction [18]. Through the normalization and transformation $\Psi_j \to \psi_j,\ x \to \sqrt{\hbar/(M\omega_{1\perp})x},\ t \to (2\pi/\omega_{1\perp})t$, Eq. (1) can be written in the form [18]:

$$\mathrm{i}\hbar\frac{\partial\psi_1}{\partial t} = \left(-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{\lambda_1^2}{2}x^2 + b_{11}|\psi_1|^2 + b_{12}|\psi_2|^2\right)\psi_1,$$

$$\mathrm{i}\hbar\frac{\partial\psi_2}{\partial t} = \left(-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{\lambda_2^2}{2}x^2 + b_{21}|\psi_1|^2 + b_{22}|\psi_2|^2\right)\psi_2,$$

where $\lambda_j = \omega_{jx}/\omega_{j\perp}$, b_{jj} and $b_{j,3-j}$ are related to $\int|\psi_j|^2 dx$, trap frequencies in the radial directions $\omega_{j\perp}$, and interactions between atoms g_{jj} or inter-component $g_{j,3-j}$ $(j = 1,2)$ [19]. For the harmonic potential, the soliton states are studied widely [6,10,15,18]. In this form, the Scarf-II potential can be considered in the Gross-Pitaevskii equations.

2.3. Parity-time-symmetry

In our traditional cognition, only the Hermitian Hamiltonian system has a full real spectrum. Put forward by Bender and his coworker in 1998, parity-time- (PT-) symmetry behaviors have attracted much attention in both non-Hermitian Hamiltonian systems and nonlinear wave systems [20]. PT-symmetry can make non-Hermitian Hamiltonian systems with complex-valued potentials possibly support fully real linear spectra and stable nonlinear modes [20,21]. That is, the potential function $U(x) = V(x) + \mathrm{i}W(x)$ satisfies $V(x) = V(-x)$ and $W(-x) = -W(x)$ [21]. PT-symmetry extends the traditional quantum theory to complex phase space and leads to numerous developments in several research fields that include nonlinear optics and BEC.

2.3.1. Parity-time-symmetric Potential

Over the past twenty years, various PT-symmetric potentials have been introduced into the nonlinear non-Hermitian Hamiltonian systems. In particular, in nonlinear Schrödinger equations with complex PT-symmetric Scarf-II potentials, periodic potentials, Gaussian potentials, harmonic potentials, and δ-signum potentials, the existence of different nonlinear local modes is analytically and numerically investigated [22-24]. In optics, the stable propagation of signal can be obtained when the propagation constant of the light is in the real spectrum range, which requires the gain-and-loss distributions in the medium to be precisely balanced to ensure the relation $n(x) = n^*(-x)$ [25,26]. The introduction of the PT-symmetry concept into optical systems leads to the discovery of many stable solitons, in which PT-symmetric potential is realized by the complex refractive index $n(x) = n_R(x) + \mathrm{i}n_I(x)$ [27-29]. Research on PT-symmetric potentials is significant in the development of BECs and optics.

2.3.2. Scarf-II Potential

Scarf-II potential, also called Gendenshtein potential. It is an important external potential in nonlinear physics [30]. Scarf-II potential is in the form of a hyperbolic function [30]. Scarf-II potential can ensure that there is an accurate solution in the Schrödinger model and can describe many physical phenomena [30,31]. The form of Scarf-II potential is

$$\mathrm{U(x) = Vsech^2(x) + iWsech(x)tanh(x).}$$

2.4. Solitons in Bose-Einstein Condensates

The soliton, as a fundamental excitation of the atomic matter waves, is studied intensively and attracted more and more attention in BECs. Various aspects of the dynamics of nonlinear matter-wave solitons within BECs are being explored in research [4,32]. This includes investigations into phenomena such as spontaneous symmetry-breaking symbiotic solitons, vortex dynamics, interference patterns, domain walls, and four-wave mixing [33]. Solitons within BECs are scrutinized under changing parameters, encompassing time-dependent atomic wave scattering lengths affected by Feshbach resonance, dynamic fluctuations in gain or loss from the thermal cloud, and the impacts of diffractive variables [34]. Many novel phenomena are discovered in multi-component BECs, including symbiotic solitons, soliton trains, soliton pairs, multi-domain walls, and multi-mode collective excitation [25]. However, there are still few studies on solitons in two-component BECs, and there are few reports on the special physical phenomena in existing studies [11]. What are the conditions for the existence of stable solitons in two-component BECs?

2.5. Solitons in Parity-time-symmetric System

Based on the literature review. The PT-symmetric potential seems can be introduced to the system to answer this question [20]. Due to the existence of PT-symmetry characteristics, stable solitons are more likely to exist in nonlinear systems. With the development of symbolic computation and soliton theory, various types of soliton solutions in parity-time-symmetric systems are analyzed in detail, including multi-peak solitons, exploding solitons, two-dimensional vortical solitons, lattice solitons and peakons [35,9,7,8]. Stable nonlinear modes are obtained by constructing BEC models with PT-symmetric potential [21]. Therefore, the PT-symmetric potential can be introduced to coupled Gross-Pitaevskii equations, and the stable solitons can be obtained in two-component BECs [4]. The solitons can be anticipated that stably exist in two-component BECs with PT-symmetric potential.

3. Research Questions

What are the system parameters for the existence of stable soliton in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential? Based on the above analysis, the coupled Gross-Pitaevskii equations with complex PT-symmetric potentials will be investigated:

$$\mathrm{i}\frac{\partial\psi_1}{\partial t} = \left(-\frac{\partial^2}{\partial x^2} - a_1(|\psi_1|^2 + |\psi_2|^2) - U_1(x)\right)\psi_1,$$

$$\mathrm{i}\frac{\partial\psi_2}{\partial t} = \left(-\frac{\partial^2}{\partial x^2} - a_2(|\psi_1|^2 + |\psi_2|^2) - U_2(x)\right)\psi_2,$$

where a_j represent the assumed-equal intra-component and inter-component interactions, $U_j(x)$ are the complex PT-symmetric potentials, and the imaginary parts of $U_j(x)$ stand for the gain-or-loss term from the thermal cloud. It not only helps researchers reveal the interval physical principles of the systems from the physical level and make reasonable explanations for mathematical conclusions but also helps researchers predict physical phenomena or results through mathematical conclusions.

4. Research Methods

Due to the complexity of nonlinear partial differential equations, traditional analytical solutions, e.g., Hirota bilinear method, Lax representation, Bäcklund transformation, and Darboux transformation, are difficult to apply [1]. In this paper, for the nonlinear Schrödinger equation, modified squared-operator and fourth-order Runge-Kutta methods will be used to investigate the dynamic behavior and stability of solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential [1-3]. The modified squared-operator method will be used for numerical solving while the fourth-order Runge-Kutta method will be used to simulate the evolution of solitons over time by the fast Fourier transform [2,3]. What is more, these two methods are widely used and adopted by researchers, and such methods are also the algorithms with better convergence and stability in numerical solutions.

5. Research Procedures

Our research procedures are divided into six steps, as shown in the table below:

Step	Tasks	Due date
1	Exploring the model's physical background.	Feb. 2024
2	Modifying algorithm code.	Apr. 2024
3	Adjusting the parameters of the system.	June 2024
4	Using algorithms for numerical solutions.	Aug. 2024
5	Verifying the stability of the numerical solutions.	Oct. 2024
6	Concluding the results.	Dec. 2024

First, we will further survey the physical background of Bose-Einstein condensates with PT-symmetric potentials. Second, we will change the model in the existing algorithm to the coupled Gross-Pitaevskii equations with complex PT-symmetric potentials. In this process, the effects of binary coupling need to be considered, that is the two equations need to be iterated at the same time. Third, we will adjust the parameters of the system based on the physical mechanism. Linear stability analysis is used to find some stable parameter regions. Fourth, based on the stable region found in the previous step, the relationship curve of stable soliton power with parameters is given through modified squared-operator methods. Fifth, the numerical solitons are evolved to verify their stability through the fourth-order Runge-Kutta method. Finally, the results will be concluded.

6. Conclusion

In this work, modified squared-operator and fourth-order Runge-Kutta methods will be carried out to investigate the dynamic behavior and stability of solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential. It is expected to give the system parameters interval range for the presence of stable solitons in two-component Bose-Einstein condensates with parity-time-symmetric Scarf-II potential.

The parity-time-symmetric potential is introduced into two-component Bose-Einstein condensates model, which proves that there may be stable nonlinear modes in two-component Bose-Einstein condensates system under the regulation of parity-time-symmetric Scarf-II potential. This work provides a new idea for the rational use of external potential control to obtain stable nonlinear modes, which can help researchers predict new physical phenomena and further guide practice.

When a research project is carried out in the above methods, the following problems exist: Due to the complexity of nonlinear partial differential equations and numerical simulations carried out in the investigation, there will be errors in numerical calculations. The calculation process may be slow, and the amount of computation required is large. The problem of large computation will be overcome by optimizing the algorithm structure.

In addition, we can consider other PT-symmetric potentials in the coupled Gross-Pitaevskii equations. Due to the limitations of the parameters in this model, the amplitudes of the two solutions are constrained by a certain relationship. Therefore, we can also consider the case of unequal intra-component and inter-component interactions.

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Appendix: Grading Sheet of Research Proposal

Class: SID No. Name: Grading:

ITEM	POINTS	Marks
Professional Appearance (10%)		
Cover page, Contents, Paging	5	
Formatting	5	
Writing Quality (15%)		
Grammar, spelling and punctuation	5	
Words, professional terms and concepts	5	
Appropriate paragraphing	5	
Quality of Analysis (65%)		
Critical Literature review		
Focus	5	
Reading	5	
Relevance	5	
Coherence	5	
Argument	5	
Research methods and procedure		
Focus	5	
Relevance	5	
Coherence	5	
Research design	5	
Presentation of information/data		
Mechanics	5	
Expression	5	
Structure	5	
Unity	5	
Citation and reference (10%)		
Citation	5	
References	5	
Total	100	